Does rotation of the interacting masses limit experiments to test the Universality of Free Fall and the Weak Equivalence Principle?

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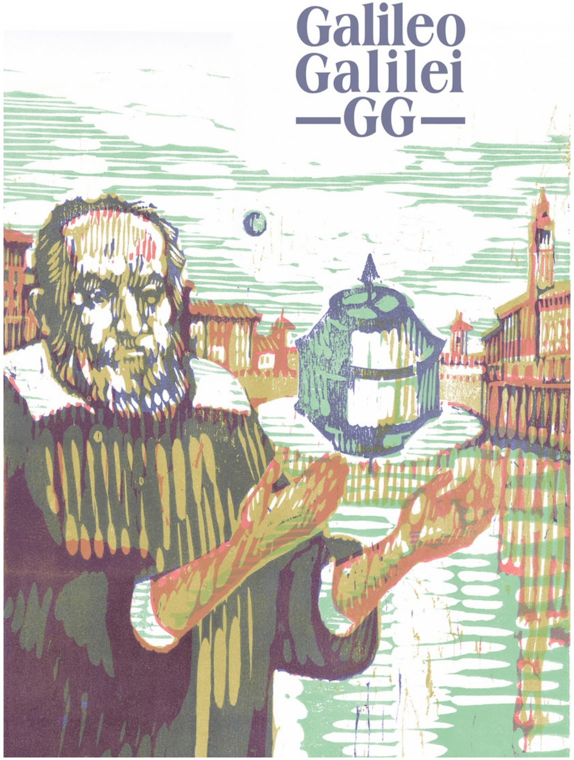
The issue

GG evaluated in 2015 within ESA M4 competition by SARP - Science Assessment Review Panel

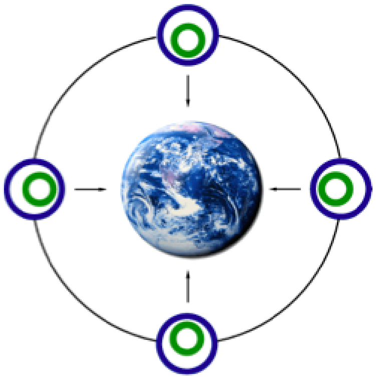
- One ESA question to SARP: *Are there any issues not mentioned in the proposal that could hamper the proposed scientific return?*

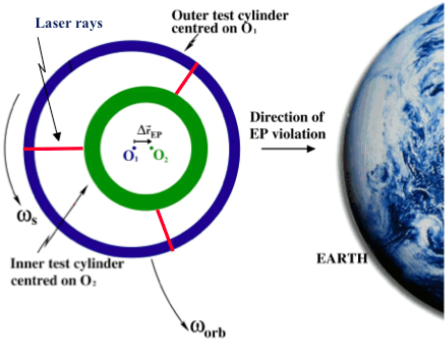
SARP: “The breakdown of the WEP is sought in the frame work of the response of test matter to terrestrial Newtonian gravitation. The source of terrestrial Newtonian gravitation is independent of the Earth’s (non uniform) rotation. Furthermore, the test cylinders in the proposed experiment are spinning. In General Relativity the gravitational field of a spinning source depends on its spin. Also the mass centroid motion of extended spinning test matter in an external gravitational field may depend on its spin and still be geodesic (independent of inertial mass) when its spin is zero. The estimates, based on General Relativity, of the effect of the Earth’s rotation on the motion of each spinning cylinder or the laser interferometer and their relevance to the interpretation of any non null signal at the expected level of accuracy have not been sufficiently explained to the satisfaction of the SARP.”



GG (“Galileo Galilei”): a mission to test the founding pillar of GR to 10^{-17} and beyond

GG design: the key features

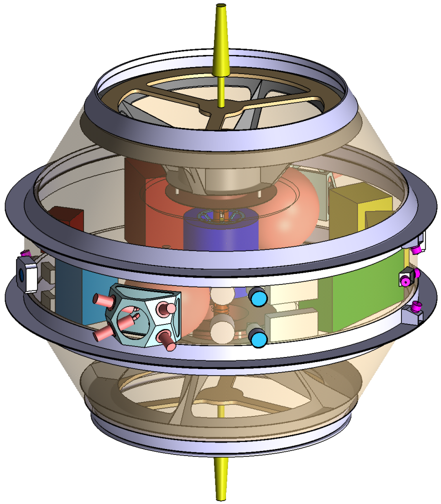




A 2D harmonic oscillator made of concentric test cylinders (very weakly coupled) in orbit around the Earth.
Violation signal is at the orbital frequency
Spin around the symmetry axis up-converts it to the much higher spin frequency (1 Hz) away from high thermal noise making integration time very short $\Rightarrow$ 1 measurement to 10^{-17} in 1 d (15 orbits); plenty of time for checking systematics

Pegna et al., PRL 2011

Nobili et al., PRD 2014



Rotation around symmetry axis, and sensitivity in the plane $\perp$ to it, respect physical symmetry and allow passive s/c stabilization by 1-axis rotation at 1 Hz.
2D sensitivity ensures:
a) rotation above the very weak coupling frequency (would be unstable in 1D)
b) that the up-converted signal is not attenuated (as it is in 1D)
A second equal composition accelerometer can be concentric too
Full size prototype on ground

Nobili et al., CQG 2012

Rotation in experiments to test UFF/WEP

1918 - Lense & Thirring

L&T (1918) paper on: *The influence of the self-rotation of central bodies on the movements of the planets and the moon according to Einstein's theory of gravitation*

"In the Newtonian theory one can exactly replace the field in the space surrounding a (stationary or rotating) sphere of uniform density with the field of a material point of equal mass. Also, according to Einstein's theory the field of a resting sphere of incompressible fluid is equivalent to that of a point mass; but for a rotating sphere this is not the case."

They write the equations of motion of a point mass in the field of a rotating spherical central body and compute the secular effect on the orbital elements node, pericenter, mean longitude (semimajor axis is unaffected; no secular effects on e, i)

None of these effects is relevant in current equivalence principle tests by LLR - to 10^{-13}

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1960 - 1961: The theoretical bases of GP-B

In 3 papers Leonard Schiff solves the problem of the motion of a gyroscope according to Einstein's theory of gravitation (the central body is rotating and the mass of the gyroscope is negligible) and makes the case for GP-B mission

In still another paper (Schiff AJP, 1960) Schiff takes Eötvös tests of WEP $\sim 1900 +$ **(Special Theory of Relativity) as best evidence by far also for deflection of light and gravitational redshift, hence he argues against a gravitational redshift mission (GP-A) and wants to test GR by measuring its direct effect on precession of a spinning body**

Schiff, PRL 1960
Schiff, Proc. Nat. Acad. Sc., 1960
Schiff, Proc. Conference "Experimental Tests of Theories of Relativity", Stanford 1961

... a lot of work was done before that Schiff could rely upon:
Lense & Thirring, Phys. Z, 1918
Mathisson, Acta Phys. Pol. , 1937
Papapetrou, Proc. R. Soc. London, 1951
Corinaldesi & Papapetrou, Proc. R. Soc. London, 1951

1970s: The binary pulsar and the full equations of motions of interacting bodies

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Neutron star binary systems and the binary pulsar PSR1913+16 require the equations of motion within GR for 2 interacting bodies with arbitrary masses, spins (even quadrupole mass moments)

Barker & O'Connel, PRD 1970

Barker & O'Connel, PRD 1975:
"Gravitational two-body problem with arbitrary masses, spins, and quadrupole mass moments"

GR effects on the acceleration of Microscope/GG test masses (I)

- Interaction of Earth's spin angular momentum $S_\oplus$ with TM orbital angular momentum (Spin-Orbit interaction): effect on **radial component** of TM acceleration:

$$a^r_{S_\oplus,L_{TM}} \sim \frac{GM_\oplus}{c^2 r^3}\frac{S_\oplus}{m_\oplus} v_{orb} \sim 10^{-10}\,\mathrm{m/s^2} \qquad (\mathrm{r} \simeq 7000\,\mathrm{km})$$

Only the radial differential acceleration competes with the signal:

$$\Delta a^r_{S_\oplus,L_{TMs}} \lesssim a^r_{S_\oplus,TM}\frac{5}{2}\frac{\Delta r_{TMs}}{r} \qquad (g(h) \simeq 8\,\mathrm{m/s^2})$$

Microscope: $\Delta r_{TMs} \simeq 0.1\,\mu\mathrm{m}$ (recovered from data analysis; $20\,\mu\mathrm{m}$ by construction):

$$\frac{\Delta a^r_{S_\oplus,L_{TMs}}}{g(h)} \sim 4.5\cdot 10^{-25} = 4.5\cdot 10^{-10}\,\eta_{Microscope}$$

GG: $\Delta r_{TMs} \simeq 10^{-9} m$ (from damping of whirl at coupling frequency before data taking sessions; self-centering in the rotating system to few tens of pm by physical laws in super-critical rotation; $10\,\mu\mathrm{m}$ requirement by construction/mounting)

$$\frac{\Delta a^r_{S_\oplus,L_{TMs}}}{g(h)} \sim 4.5\cdot 10^{-27} = 4.5\cdot 10^{-10}\,\eta_{GG}$$

GR effects on the acceleration of Microscope/GG test masses (II)

- Interaction of Earth's spin angular momentum $S_\oplus$ with TM spin angular momentum S_{TM} (Spin-Spin interaction): effect on **radial** acceleration of one TM

$$a^r_{S_\oplus S_{TM}} \sim \frac{G S_\oplus S_{TM}}{c^2 r^4 m} \sim 2 \cdot 10^{-21} \cdot \frac{S_{TM}}{m}\ \mathrm{ms}^{-2}$$

Largest effect in GG because of $\nu_{spin} = 1\,\mathrm{Hz}$; $S_{TM}/m \lesssim 0.09\,\mathrm{m}^2\mathrm{s}^{-1}$
Differential acceleration dominated by difference in S_{TM} between test masses, amounting to $\Delta S_{TMs}/m \simeq 0.06\,\mathrm{m}^2\mathrm{s}^{-1}$

$$\frac{\Delta a^r_{S_\oplus S_{TMs}}}{g(h)} \sim 1.5 \cdot 10^{-23} \sim 1.5 \cdot 10^{-6}\,\eta_{GG}$$

Even less relevant in Microscope (smaller spin rate and larger WEP target)

Effect of $S_\oplus$, S_{TM} on orbit precession

$$\Omega_{S_\oplus S_{TM}} \sim \frac{3}{2}\frac{GS_\oplus}{c^2 r^5 \omega_{orb}} \cdot \frac{S_{TM}}{m}$$

Differential part dominated by difference in S_{TM} between test masses:

$$\Delta\Omega_{S_\oplus S_{TM}} \sim 2.8 \cdot 10^{-26}\,\mathrm{rad/s} \qquad \Rightarrow \mathrm{P} \sim 7 \cdot 10^{18}\,\mathrm{yr}$$

In one orbital period the differential displacement of the orbital nodes is 10^{-3} pm; only a smaller contribution to the differential displacement of the test masses in the radial direction is relevant (GG target displacement is 0.6 pm radial).
Even less relevant for Microscope (which in addition must larger displacements, of 10 pm)

GR effects on the spin axis of the test masses: De Sitter precession

De Sitter precession (independent of Earth's rotation):

$$\Omega_{DS} \sim \frac{c}{r}\left(\frac{GM_{\oplus}}{c^2 r}\right)^{3/2} \sim 10^{-12}\,\mathrm{rad/s}$$

The differential contribution for GG is (smaller than for Microscope because of smaller offsets):

$$\Delta\Omega_{DS} \sim \Omega_{DS}\frac{5}{2}\frac{\Delta r}{r} \sim 3.6 \cdot 10^{-28}\,\mathrm{rad/s}$$

In one orbit the resulting relative displacement between points on the spin axes (at $\sim 20\,\mathrm{cm}$ height) is $4 \cdot 10^{-25}\,\mathrm{m}$
($\sim 1\,\mathrm{nm}$ displacement in common mode along the precession cone)

GR effects on the spin axis of the test masses: Lense-Thirring precession

Lense-Thirring precession (caused by Earth's rotation):

$$\Omega_{LT} \sim \frac{GM_\oplus}{c^2 r}\frac{S_\oplus}{M_\oplus r^2}\,\mathrm{rad/s}$$

The differential contribution is due to the test masses not being perfectly centered. In GG:

$$\Delta\Omega_{LT} \sim \Omega_{LT}\cdot\frac{3\Delta r}{r} \sim 6.5\cdot 10^{-30}\,\mathrm{rad/s}$$

which is 50 times smaller than the De Sitter precesssion effect

Is rotation of the test masses an issue for the laser gauge read-out of GG?

- Laser gauge is linear $\Rightarrow$ large gaps between test masses (2.5 cm in GG for low gas damping noise and negligible patch effects) (Cap gauge $\propto 1/D \Rightarrow$ needs small gaps; 600 μm in Microscope).

  - Differential measurements with 1 pm/$\sqrt{\text{Hz}}$ @ 1 Hz noise more or less routine

  - Common mode effects at 10 nm level are not a problem

- Laser gauge does not require cryogenics like SQUIDs for STEP

- Mike Shao (JPL) realized it for SIM about 10 years ago.

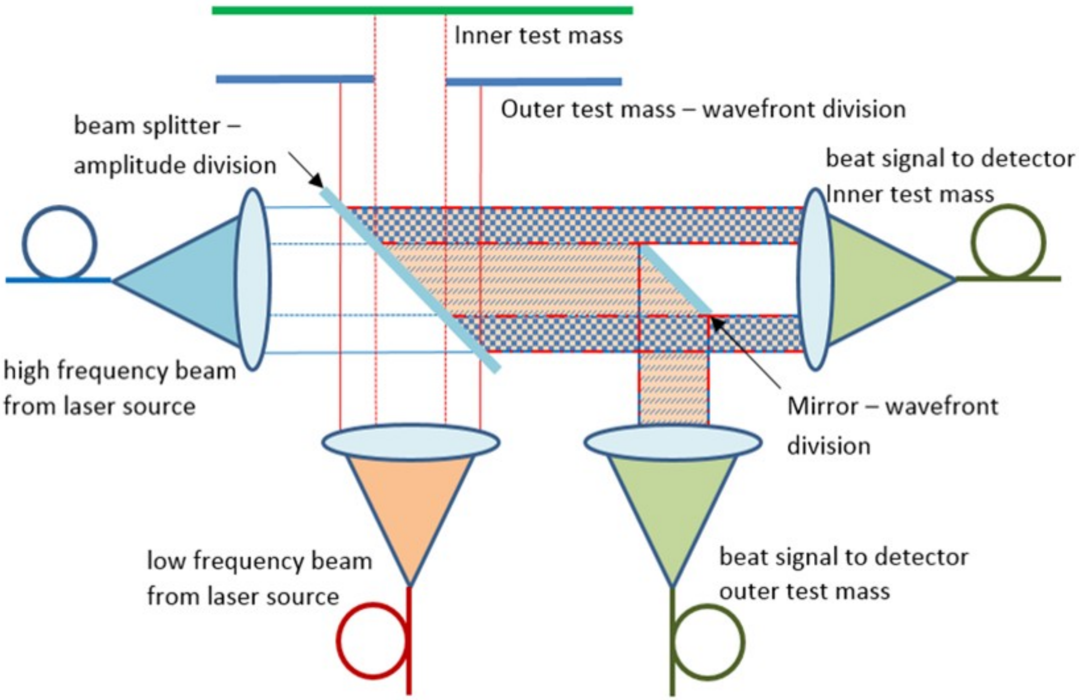
  - Heterodyne laser interferometer based on spatial separation rather than polarisation separation of the beams to reduce cyclic error (COmmon–Path Heterodyne Interferometer - COPHI)

  - 1 pm/$\sqrt{\text{Hz}}$ @ 1 Hz demonstrated up to 10 m separation (lower noise than SQUID and cap gauge)

  - Mike proposed a version for GG in 2010 in order to exploit GG low thermal noise and short integration time to separate systematics from signal (investigated during 2.5-month study of GG at JPL)

- Similar one to fly soon on LISA-PF (over 35 cm distance)

Spatially separated heterodyne laser gauge for GG

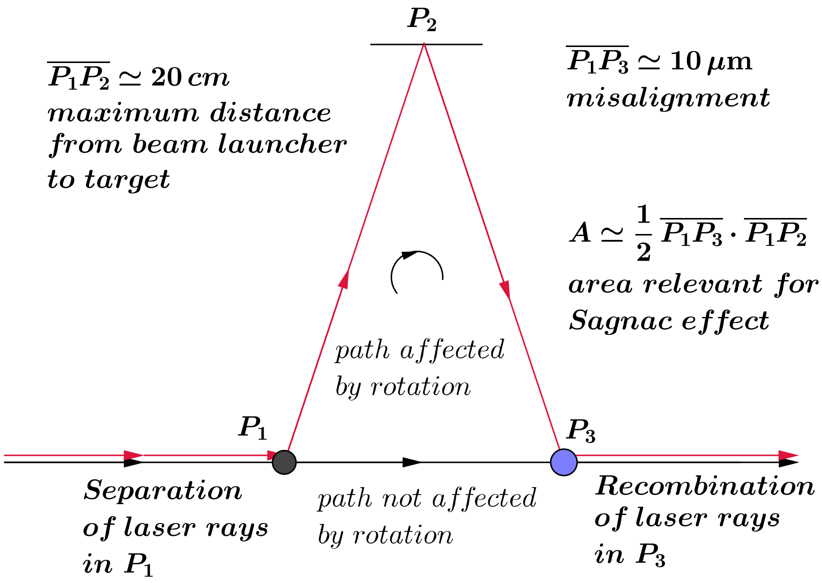


Effect of rotation on GG laser gauge.
(I) Perfect alignment

Rotation may result in a spurious displacement because of the Sagnac effect (as in a laser gyro) if the laser rays which are separated and recombined enclose a non zero area

- Perfect alignment: the area enclosed by the interfering laser rays (form separation to recombination) is zero, no spurious effect

Effect of rotation on GG laser gauge.
(II) Alignment not perfect



Spurious displacement due to Sagnac effect:

$$\Delta x \simeq \frac{\omega_{spin}}{c} 2A \simeq 6.3 \cdot 10^{-14}\,\mathrm{m}$$

2 orders of magnitude smaller than target. In addition, only time variations of the spin frequency close to the spin frequency - frequency at which the laser gauge operates - do matter. Remember: in space, no motor, no bearings, very high spin energy hence spin axis essentially unaffected by torques...

Conclusions

- Based on well established literature GR effects on the GG experiment are negligible by and large (despite the 1 Hz spin rate), and even more so in Microscope
- In GG 1 Hz rotation does not affect the laser gauge read-out (with realistic construction errors)