COLD DAMPING IN MICROSCOPE

A QUANTUM THERMODYNAMICAL ANALYSIS OF NOISE

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MICOSCOPE COLLOQUIUM IV PALAISEAU, 16 NOV 2015

Main noise source : thermal noise of mechanical damping due to 7 μm gold wire

Active damping from servo control does not increase noise

Proposal

Use Microscope as an experiment

- on thermal noise of large masses at very low frequencies
- on thermal noise active reduction

OUTLINE

The physics of cold damping

- analysis in term of thermodynamics
- ultimate limits and compatibility with quantum fluctuations

Using cold damping to improve measurements

- early modelization of a capacitive accelerometer

Proposal for Microscope

Cold damping : Active reduction of thermal noise in electro- or opto-mechanical systems.

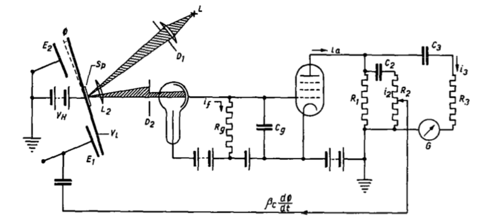
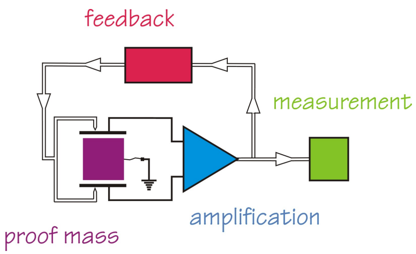
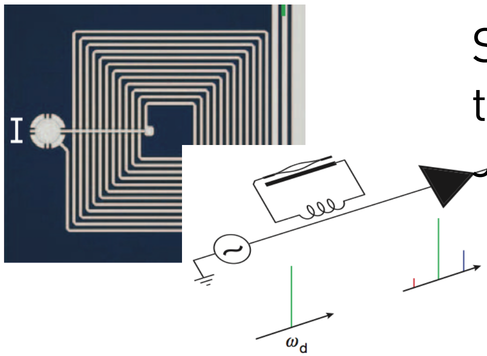


Fig. 1. Schematic arrangement of the electrometer, the damping circuit and the recording galvanometer.

The reduction in the brownian motion of electrometers, J.M.W. Milatz et al. Physica XIX, 195-207 (1953)

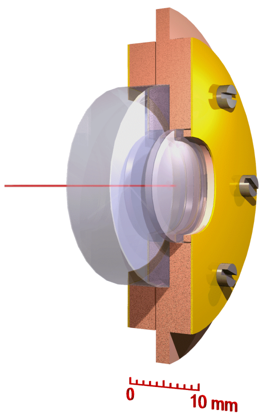


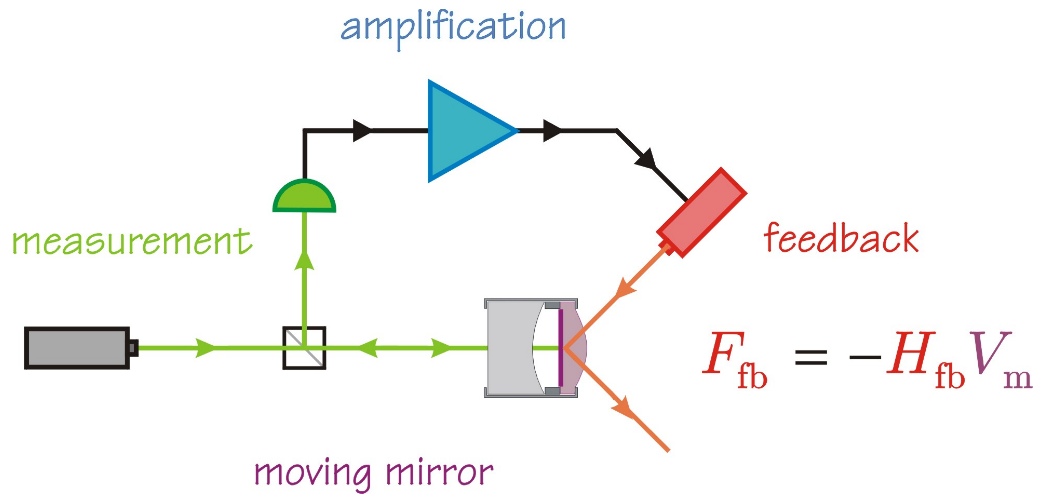
Quantum theory of fluctuations in a cold damped accelerometer. F. Grassia, J.M. Courty, S. Reynaud, P. Touboul. European Physical Journal D, 2000, 8, pp.101-110

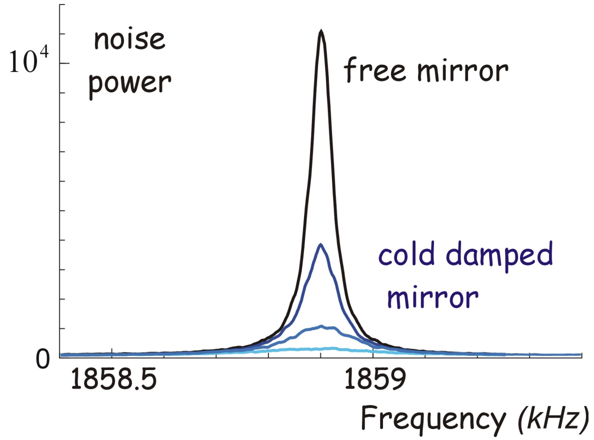


Sideband cooling of micromechanical motion to the quantum ground state
J. D. Teufel et al. Nature 475, 359–363 (21 July 2011)

COLD DAMPING IN OPTOMECHANICAL SYSTEM







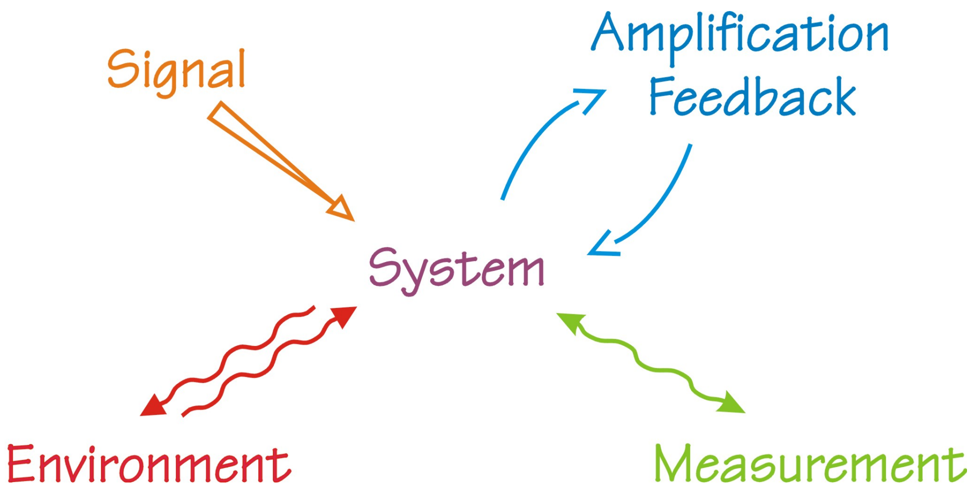
Mechanical oscillator is
an acoustic mode of a miror

Motion is measured with Fabry Perot

Force is radiation pressure of a laser

Cooling of a mirror by radiation pressure
P.F. Cohadon, A. Heidmann, M.Pinard, Phys. Rev. Lett. 83, 3174 (1999).

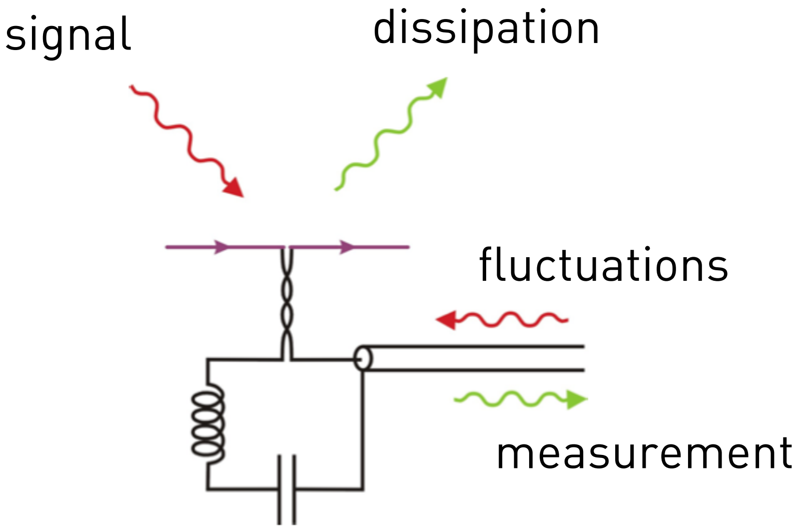
NON IDEAL MEASUREMENTS



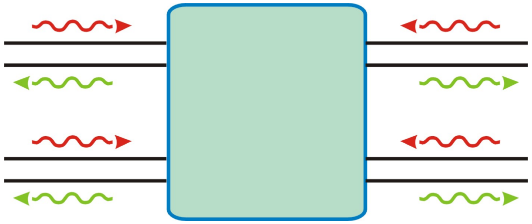
Important elements to consider in sensitivity analysis of actual measurements :

- Thermodynamical and quantum noises
- Active systems for signal amplification and servocontrols
- Spectral analysis of noise
- Modelization of complex devices

MEASUREMENT AS A SCATTERING



Fluctuation-dissipation theorem ensures consistency between the thermal noise of the oscillator and the coupled fluctuations bath

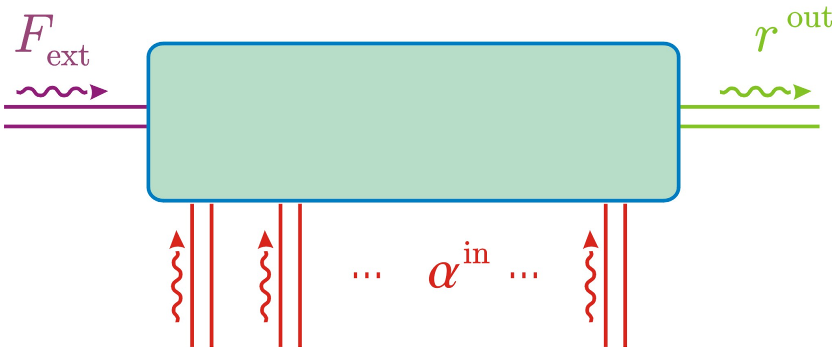


Unitarity of S matrix enforces

thermodynamic constraints

quantum constraints

Scattering of quantum fields



Force estimator

$$\hat{F}_{\text{ext}} \propto r^{\text{out}}$$

$$= F_{\text{ext}} + \sum_{\alpha} \mu_{\alpha} \alpha^{\text{in}}$$

Added noise

$$\Sigma_{FF} = \sum_{\alpha} \left|\mu_{\alpha}\right|^2 \sigma^{\text{in}}_{\alpha\alpha}$$

Thermal and quantum noises

$$\sigma^{\text{in}}_{aa}[\omega] = \frac{1}{2}\coth\frac{\hbar|\omega|}{k_B T_a}$$

$$= \frac{1}{e^{\frac{\hbar|\omega|}{k_B T_a}} - 1} + \frac{1}{2}$$

Thermal — Quantum

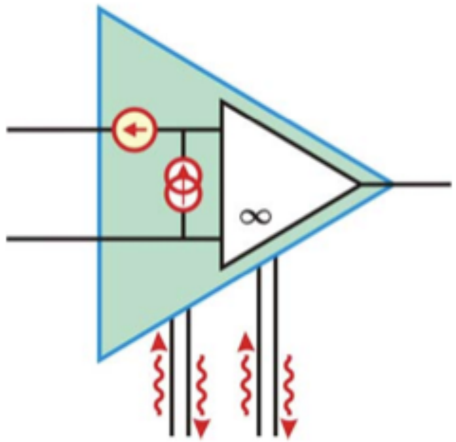
$T \to \infty \quad \hbar\,|\omega|\,\sigma_{aa} \simeq k_B T_a$

$T \to 0 \quad \hbar\,|\omega|\,\sigma_{aa} \simeq \frac{1}{2}\hbar\,|\omega|$

Amplification is also treated as a scattering.

Example : operational amplifier

Voltage noise and current noise

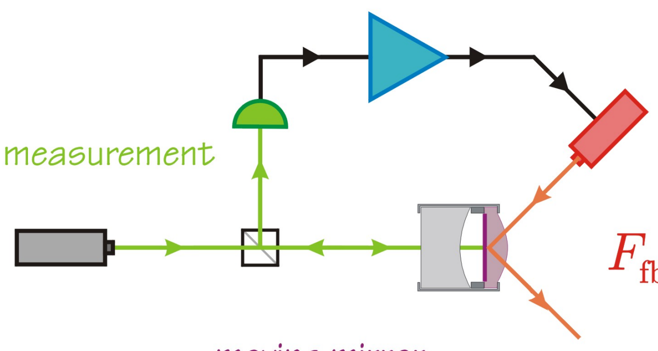


Characterized by noise impedance and noise temperature

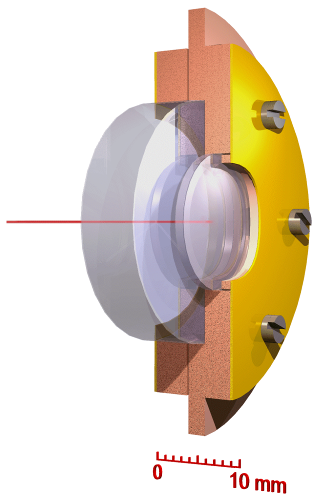
Charge and Flux are conjugated in quantum regime.

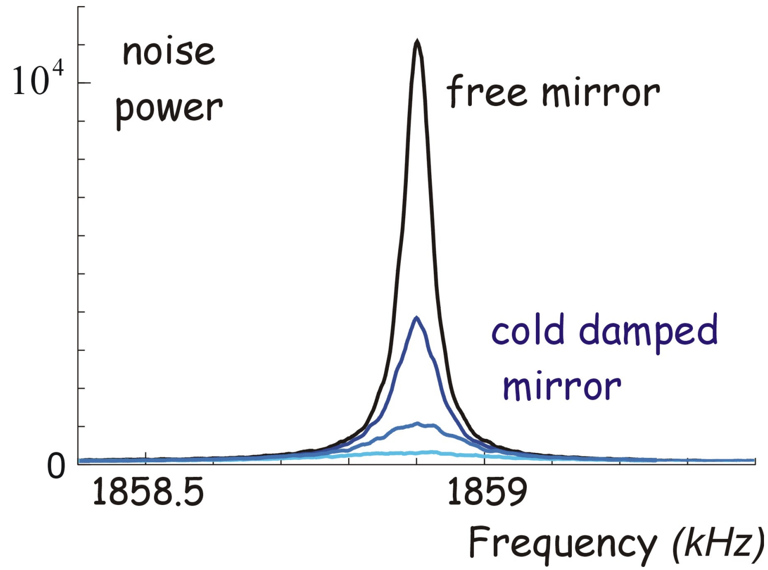
Quantum noise in ideal operational amplifiers,
Courty, Grassia, Reynaud, Europhys. Lett. 46 (1999), 31-37

COLD DAMPING IN OPTOMECHANICAL SYSTEM



$$F_{fb} = -H_{fb} V_m$$



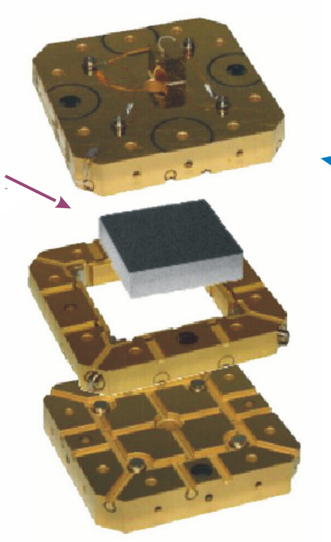


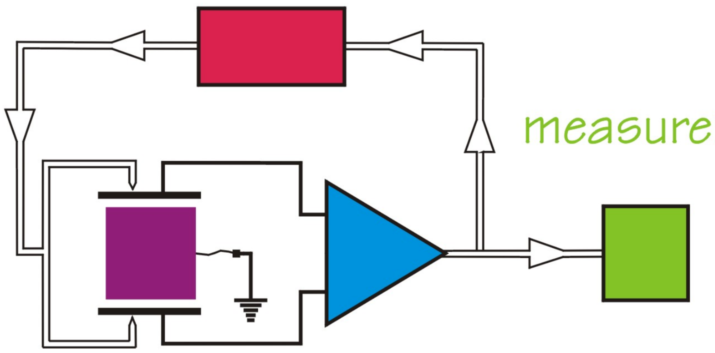
$$T = \frac{H_m}{H_m + H_{fb}} T_m$$

Cooling of a mirror by radiation pressure

P.F. Cohadon, A. Heidmann, M.Pinard, Phys. Rev. Lett. 83, 3174 (1999).

THE STAR ACCELEROMETER





Detection
- amplification noise
- back action of servocontrol

Active control of proof mass
- restoring force
- cold damping

$$\sqrt{\Sigma_{aa}} = 1.2 \cdot 10^{-12}\ m\ s^{-2} \ / \ \sqrt{Hz}$$

Quantum theory of fluctuations in a cold damped accelerometer.
F. Grassia, J.M. Courty, S. Reynaud, P. Touboul. E.P.J.l D, 2000, 8, pp.101-110

$$\hat{F}_{ext} = F_{ext} + \sum_\alpha \mu_\alpha \alpha^{\text{in}}$$

$$\mu_m = -\sqrt{2\hbar\,|\Omega|\,H_m}$$

$$\mu_{l_2} = -\frac{i\Omega\sqrt{\hbar}}{\sqrt{2R_l\omega_t\varkappa_t}}\Xi_m \qquad \mu_{l_1} = 0$$

$$\mu_{r_1} = -\frac{\Omega\sqrt{\hbar R_r}}{2\sqrt{2\omega_t}Z_f\varkappa_t}\Xi_m \qquad \mu_{r_2} = 0$$

$$\mu_{a_1} = -\mu_{b_1} = \sqrt{2\hbar R_a\omega_t}\left(-\varkappa_t + \frac{\Omega}{2\varkappa_t\omega_t Z_f}\Xi_m\right)$$

$$\mu_{a_2} = -\frac{i\Omega\sqrt{\hbar R_a}}{\sqrt{2}\varkappa_t\sqrt{\omega_t}}\Xi_m\left(\frac{1}{R_a} - \frac{1}{R_l} - \frac{1}{Z_t}\right)$$

$$\mu_{b_2} = -\frac{i\Omega\sqrt{\hbar R_a}}{\sqrt{2}\varkappa_t\sqrt{\omega_t}}\Xi_m\left(\frac{1}{R_a} + \frac{1}{R_l} + \frac{1}{Z_t}\right)$$

$$\Xi_m = H_m - iM\Omega + \frac{iK}{\Omega}$$

Characterization of all the measurement characteristics
measurement, noise, backaction, correlations

Noise of active control negligeable
low temperature of amplifier noise
ratio of signal frequency and operation frequency

Sensitivity limited by residual damping

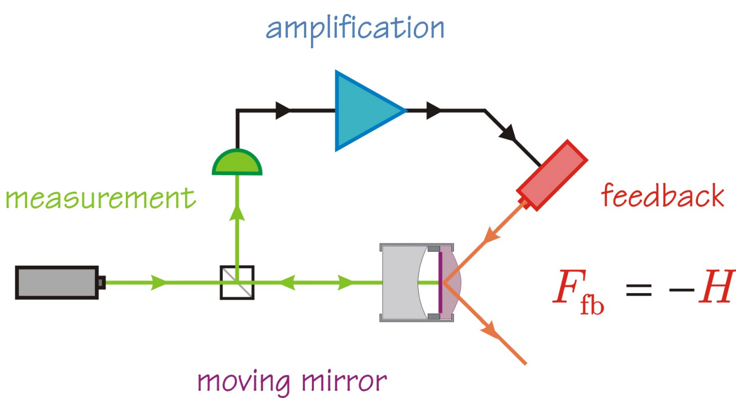
$$\Sigma_{FF} = 2\,H_m\,k_B\,\Theta_m + 4\left(r + \frac{1}{r}\right)|\Xi_m|\,\frac{\Omega}{\omega_t}\,k_B\Theta_a$$

$$r = 2\,\varkappa_t^2\,\frac{\omega_t}{\Omega}\,\frac{R_a}{|\Xi_m|}$$

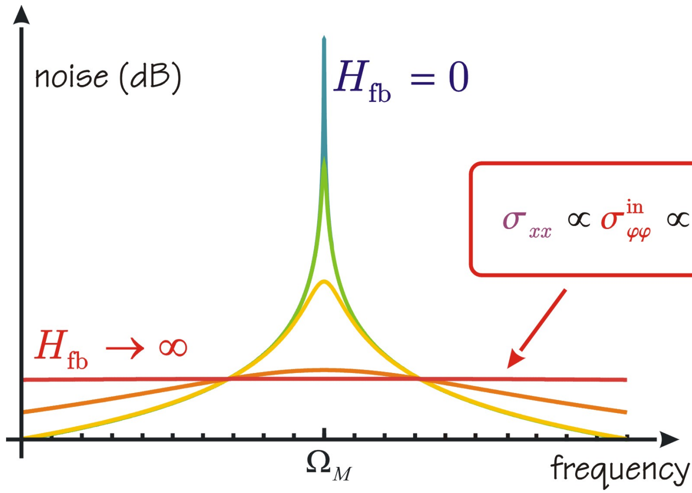
$$M = 0.27\ kg$$
$$H_m = 1.3\cdot 10^{-5}\ kg\ s^{-1}$$
$$\Theta_m = 300\ K$$
$$\Omega = 2\pi\cdot 5\cdot 10^{-4}\ Hz$$
$$\omega_t = 2\pi\cdot 10^5\ Hz$$
$$R_a = 1.5\cdot 10^5\ \Omega$$
$$T_a = 1.5\ K$$

Quantum theory of fluctuations in a cold damped accelerometer.
F. Grassia, J.M. Courty, S. Reynaud, P. Touboul. E.P.J.l D, 2000, 8, pp.101-110

QUANTUM LIMIT OF COLD DAMPING



$$F_{fb} = -H_{fb} V_m$$



$$E = \hbar\Omega_m \left(n_T \frac{H_m}{H_m + H_{fb}} + \frac{1}{2} \right)$$

Energy is reduced to ground state energy -↑ temperature is zero

J.M. Courty, A. Heidmann, M. Pinard Eur. Phys. J. D 17, 399 (2001)

PROPOSAL

Response to the call for ideas

Collaboration with Microscope Team in order to :

Perform the analysis on current sensor design

Evaluate effectiveness of cold damping

Use noise as a source of information on the instrument operation physics

Aim is noise analysis of acceleration sensors.

- Data in all the frequency bands :
  - output of the acceleration sensors
  - data from the measurement chain
  - Data from the probe mass control loop.
- Additional information such as the sensor parameters are also needed
  (gains in the measurement and in the control loop)
- some housekeeping data would also be useful,
  (temperatures for example)

MISSION IMPACT

Which sequence of the mission scenario is concerned?

All sequences of calibration as well as measurement phases

Which specific satellite orientation is needed? Or spin?

No requirement on orientation

It would be useful to have different settings for the probe masses control loops, in particular to compare situations with different values of active damping.

Dedicated sequences of calibration could be useful

MODUS OPERANDI

In which category would you classify your proposal (this choice is indicative and could evolve after discussions with the SWG): case 1 or case 2?

This proposal concerns mainly category 1
The proposer is able, after discussion with the MICROSCOPE team, to clearly define the mission data that are of specific interest and necessary for his analysis.
In that case, the SPG performs the specific data processing (that must be already sufficiently explained in the proposal) by cooperating closely with the proposer's team and delivers to the proposer the required data useful for the proposed analysis.

Additional support : This analysis work requires a close interaction with MICROSCOPE Team, for the modelisation of the accelerometer as well as the data preparation and analysis. The support for a post-doc shared between the Microscope Team and LKB would be very helpful.